



Politechnika  
Wrocławska

# Zjawiska nieliniowe w światłowodach

## W11OPA-SM0050W (FTP003030W)

### rok akademicki 2024/25

### semestr zimowy

## Wykład 7

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23rd Slovak-Czech-Polish Optical Conference, Štrbské Pleso, September 02-06, 2024

# Looking into the landscape of frequency conversion processes in optical fibers: from single mode to multimode



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05.09.2024



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# Looking into the landscape of frequency conversion processes in optical fibers: from single mode to multimode



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# Outline

## Introduction

- Description of frequency conversion processes in optical fibers

## Single mode propagation

- All-normal dispersion supercontinuum
- Soliton self-frequency shift

1

## Birefringent fibers

- Polarized all-normal dispersion SC
- Solitons - orthogonal Raman scattering

2

## Few mode fibers

- Intermodal-vectorial four-wave mixing
- Far-detuned four-wave mixing

few

## Multimode fibers

- Discretized conical emission

many





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- Far-detuned four-wave mixing

## Multimode fibers

- Discretized conical emission



# Introduction

Lasers

nonlinear  
fiber optics

frequency  
conversion  
processes

Optical  
fibers



# Introduction

## Experiment

- Laser source

wavelength, pulse/CW, duration, energy/power

- Nonlinear fiber

chromatic dispersion, effective mode area, overlap coefficients

- Detection setup

spectrometer, optical spectrum analyzer, autocorrelator, FROG system

## Theory/simulations

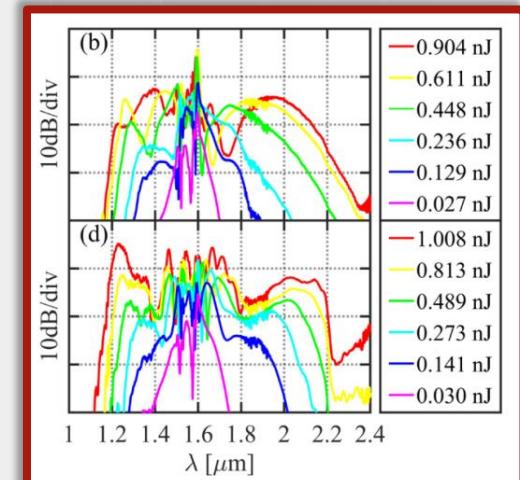
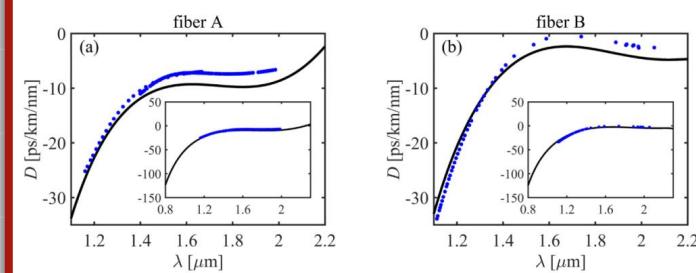
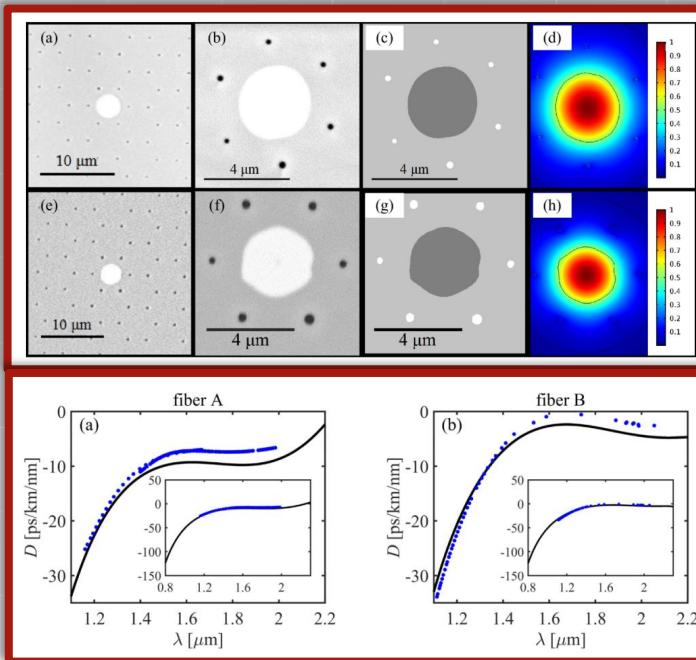
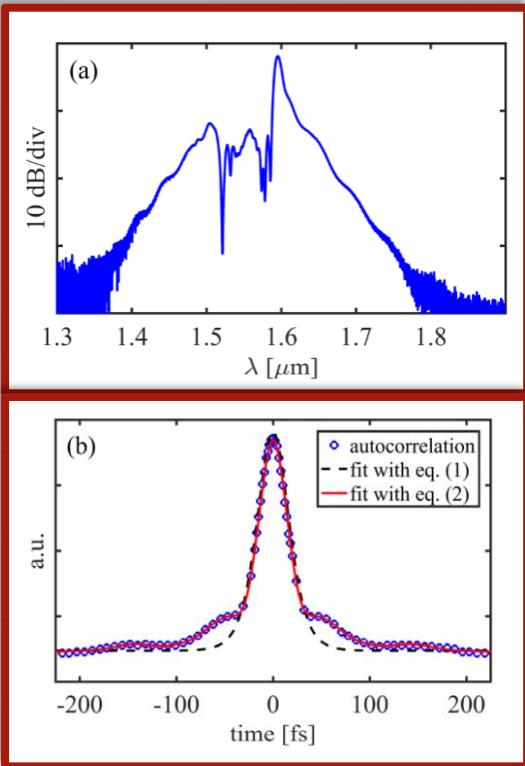
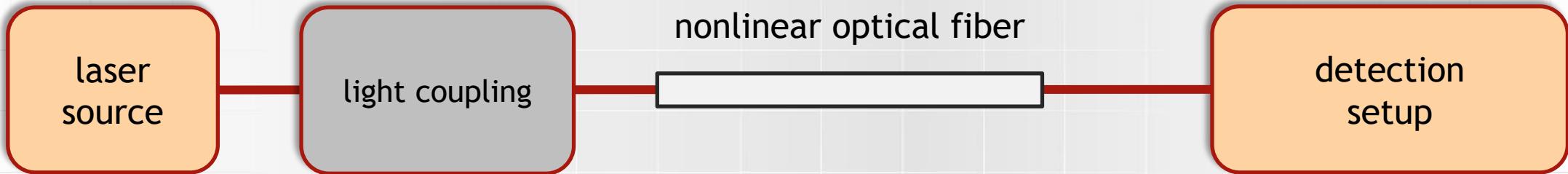
- Input field

- Fiber properties

- Output field

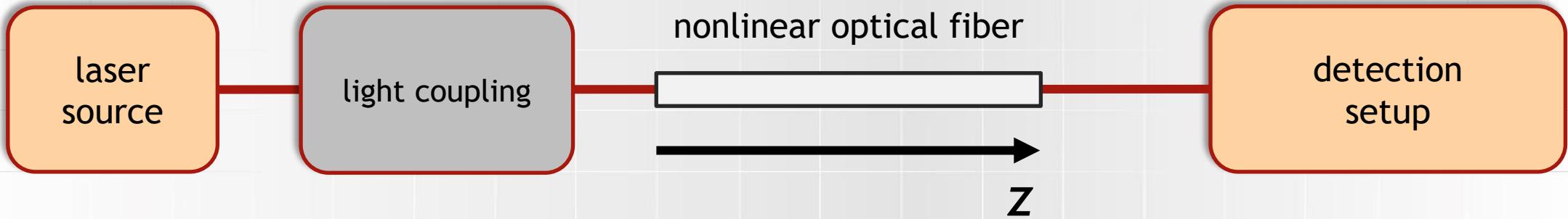


# Typical experimental setup





# Numerical experiment



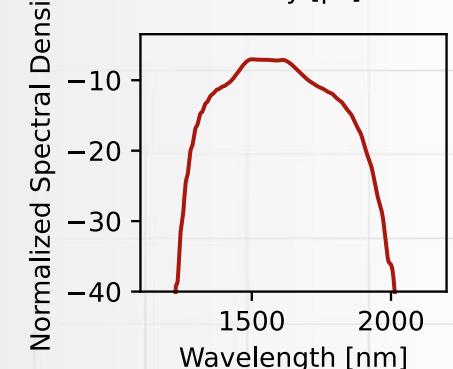
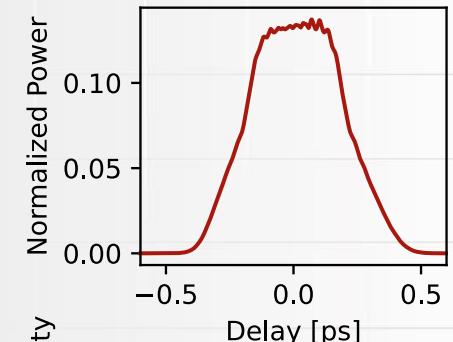
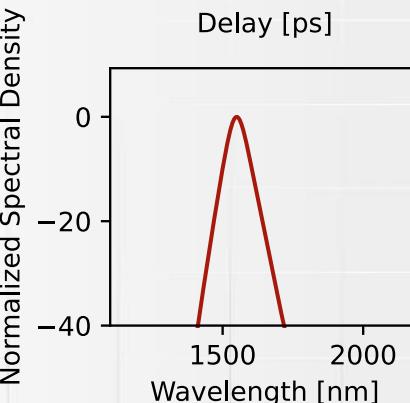
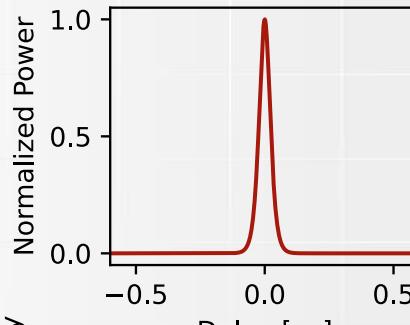
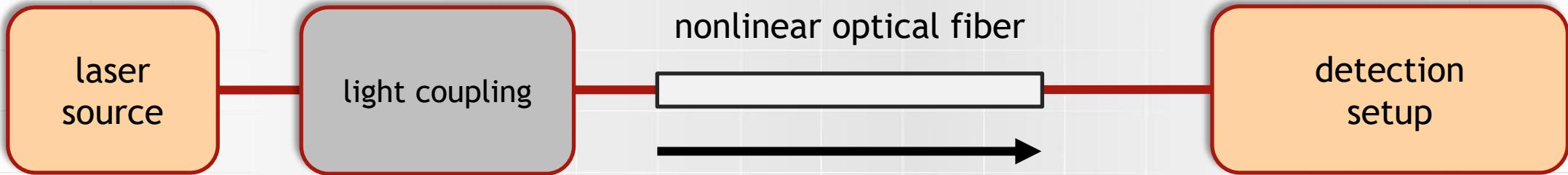
$$\begin{aligned} A(0, T) &= \mathcal{F}^{-1}\{\tilde{A}(0, \Omega)\} \\ \tilde{A}(0, \Omega) &= \mathcal{F}\{A(0, T)\} \end{aligned}$$

$$\frac{\partial A}{\partial z} = D(A) + N(A)$$

$$\begin{aligned} I(\Omega) \\ I(T) \\ S(\Omega, T) \end{aligned}$$

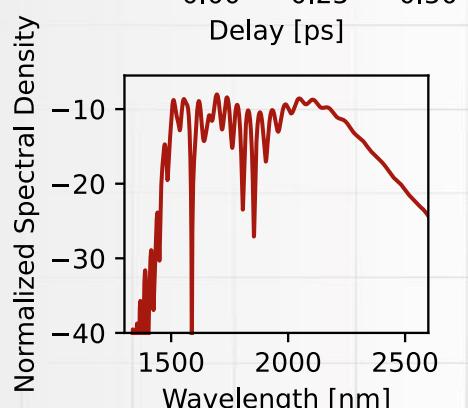
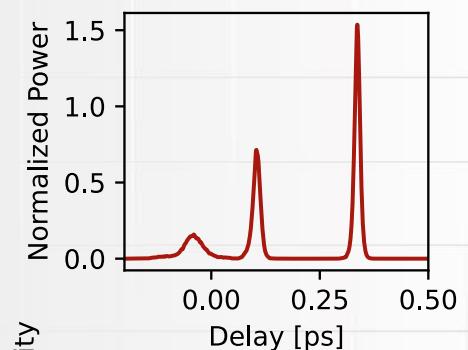
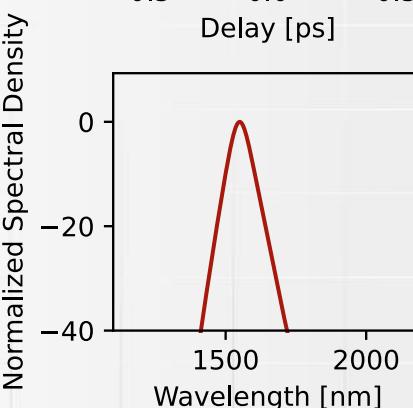
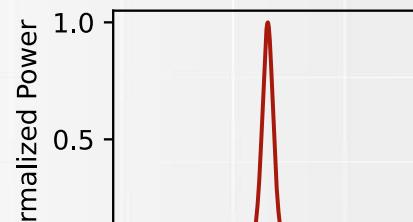
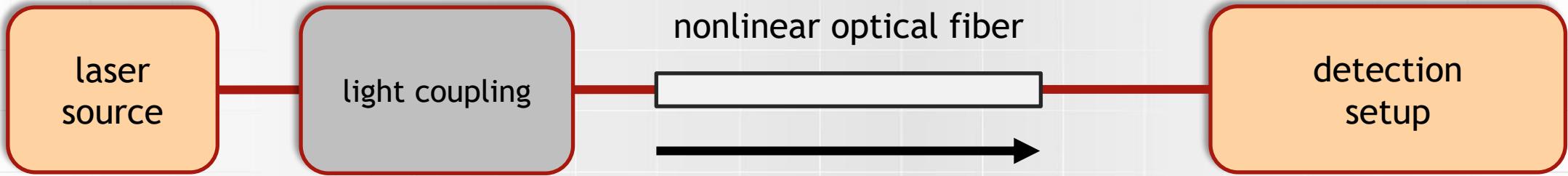


# Numerical experiment

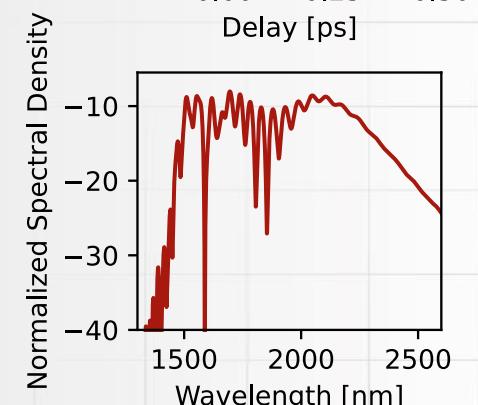
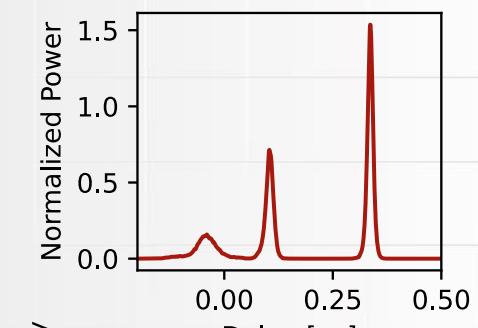
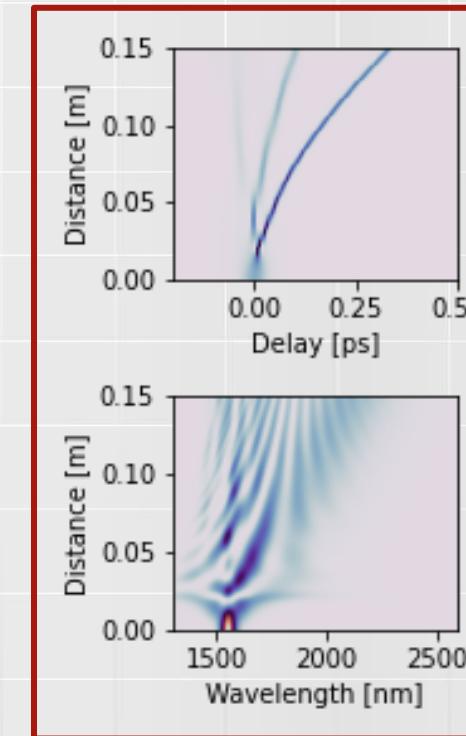
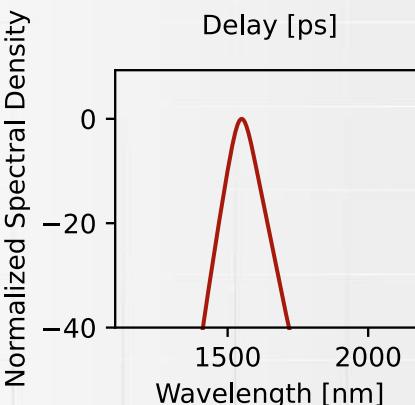
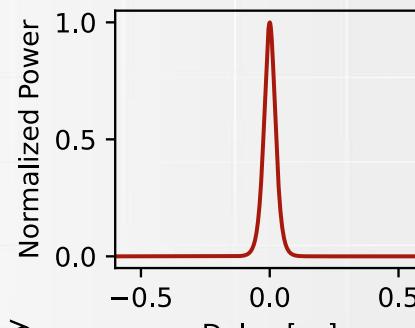
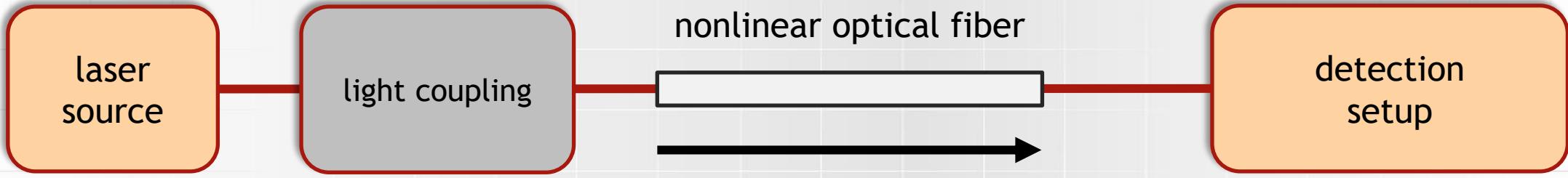


normal  
dispersion  
fiber

# Numerical experiment

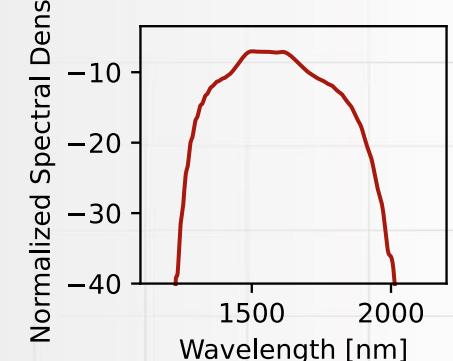
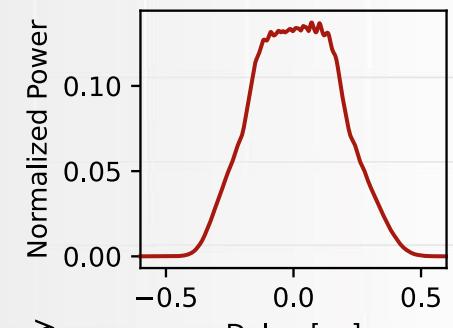
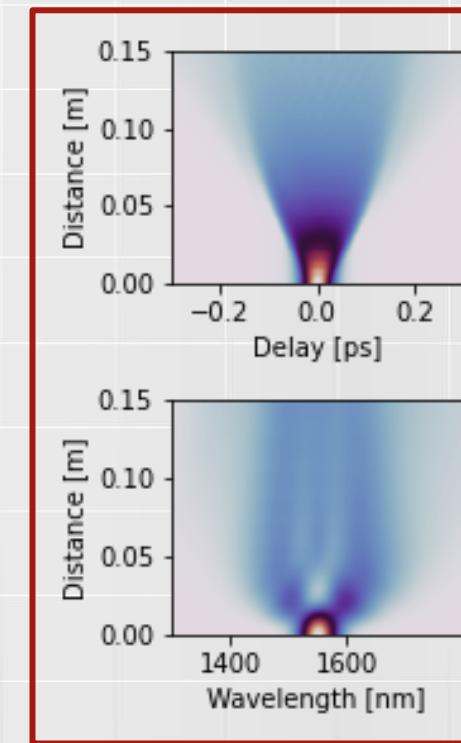
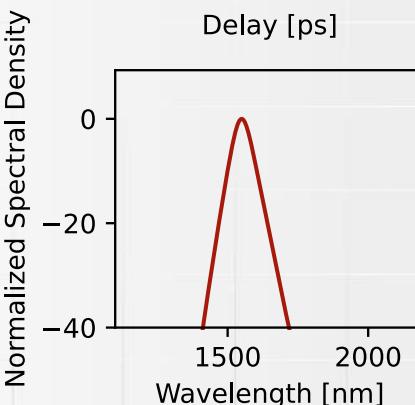
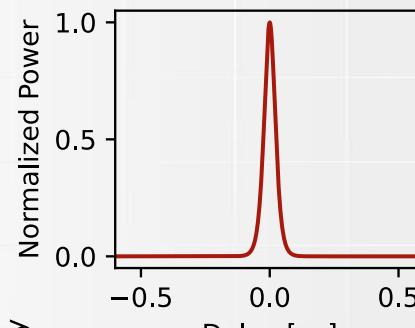
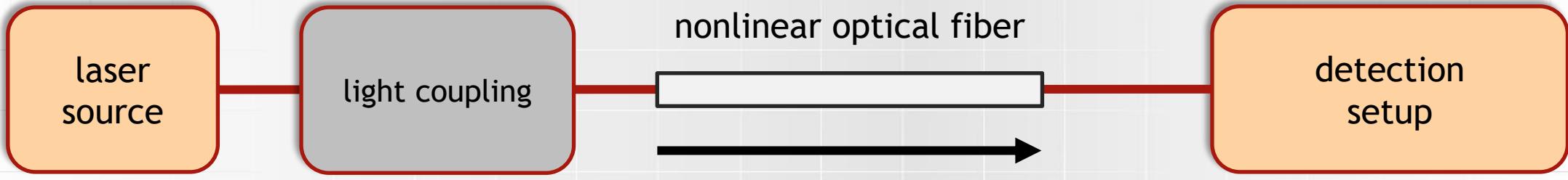


# Numerical experiment





# Numerical experiment





# Nonlinear Schrödinger equation

Generalized nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} = \underbrace{-\frac{\alpha}{2} A + i \sum_{n=1}^{\infty} \frac{i^n \beta_n}{n!} \frac{\partial^n A}{\partial t^n}}_{D(A)} + \underbrace{i \gamma A \int_0^{\infty} R(t') |A(z, t-t')|^2 dt'}_{N(A)}$$

Nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} = \left( -\frac{i \beta_2}{2} \frac{\partial^2}{\partial t^2} + i \gamma |A|^2 \right) A$$

$$i \hbar \frac{\partial}{\partial t} \Psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \Psi$$



# Frequency conversion processes

## Optically induced change in the refractive index

- self-phase modulation (SPM)
- cross-phase modulation (XPM)
  - same mode - different wavelengths
  - same mode - orthogonal polarizations
  - different modes
- four-wave mixing (FWM)
- modulation instability (MI)

## Inelastic scattering

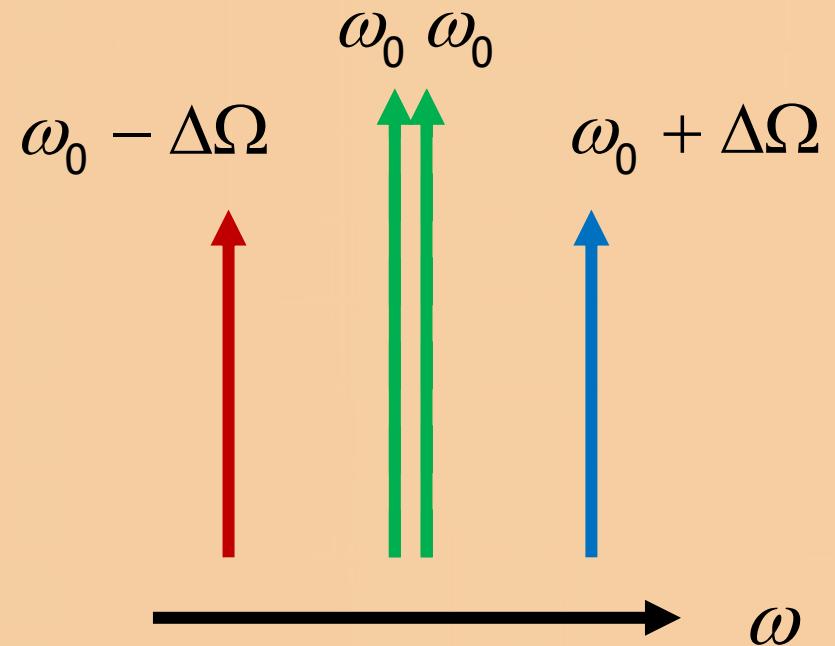
- stimulated Raman scattering (SRS)



# Frequency conversion processes

Optically induced change in the refractive index

- SPM
- XPM
- MI
- degenerated FWM





# Frequency conversion processes

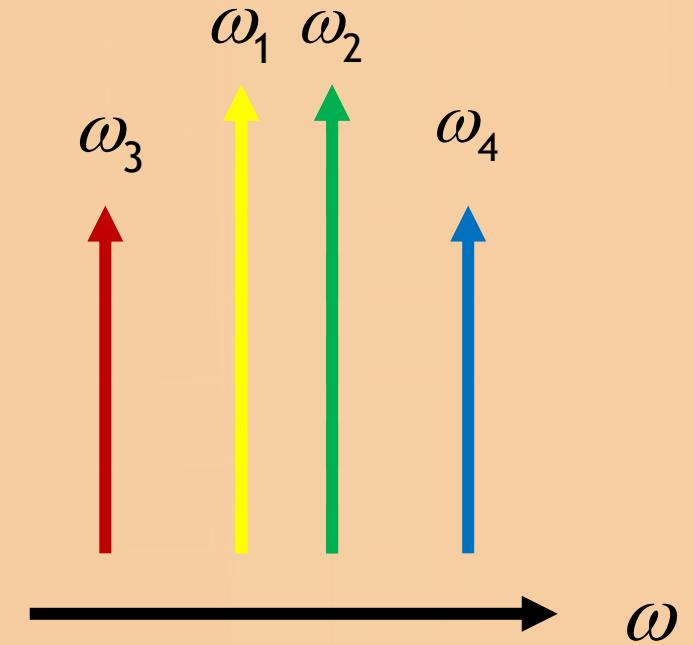
Optically induced change in the refractive index

- four-wave mixing

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

$$\beta_1 + \beta_2 = \beta_3 + \beta_4$$

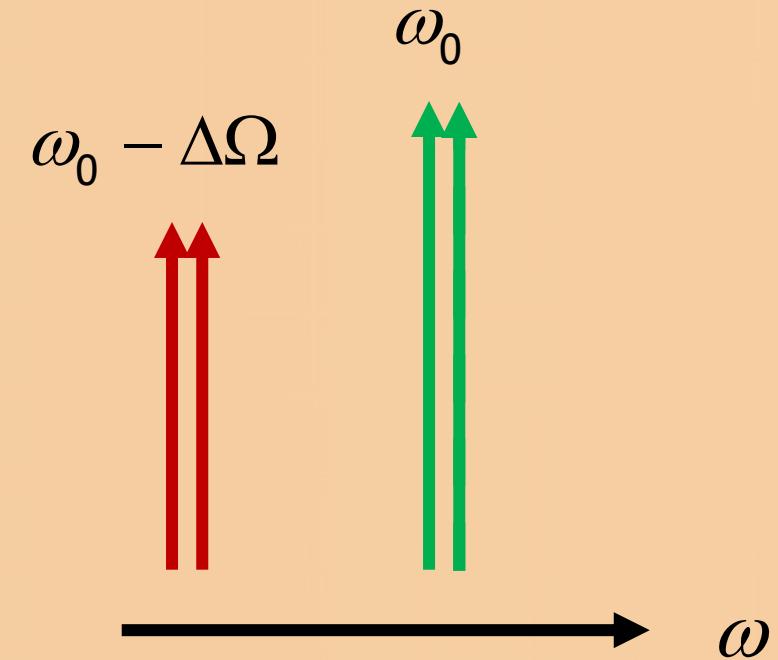
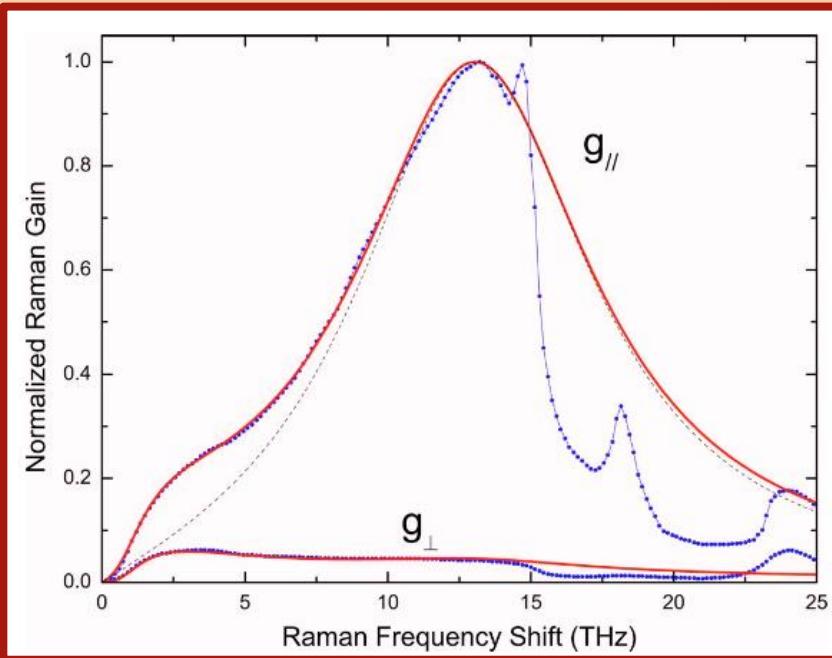
$$\beta_1 + \beta_2 = \beta_3 + \beta_4 + \Delta k_{NL}$$



# Frequency conversion processes

## Inelastic scattering

- stimulated Raman scattering (SRS)



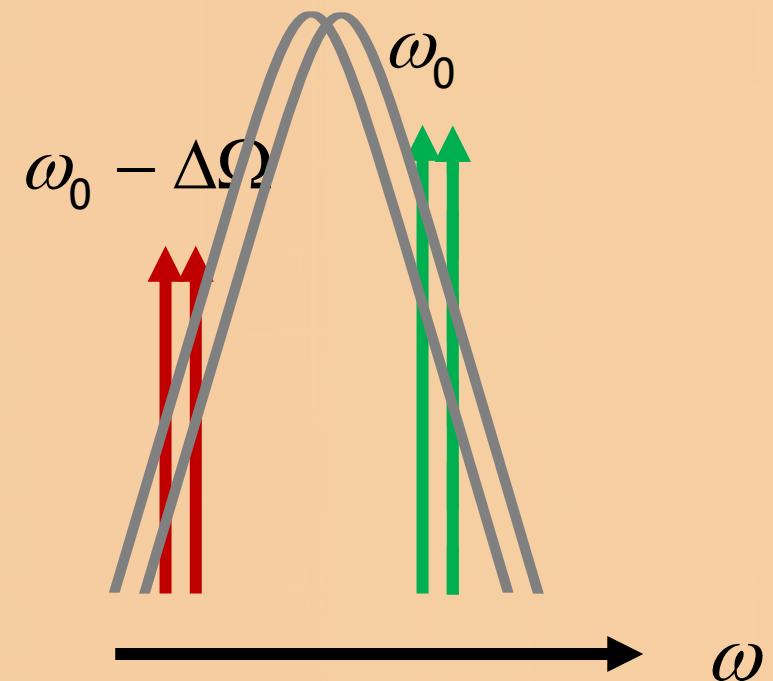
Q. Lin, G. P. Agrawal, Optics Letters, 31(21): 3086 (2006)



# Frequency conversion processes

## Inelastic scattering

- Intrapulse Raman scattering (SRS)





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## Multimode fibers

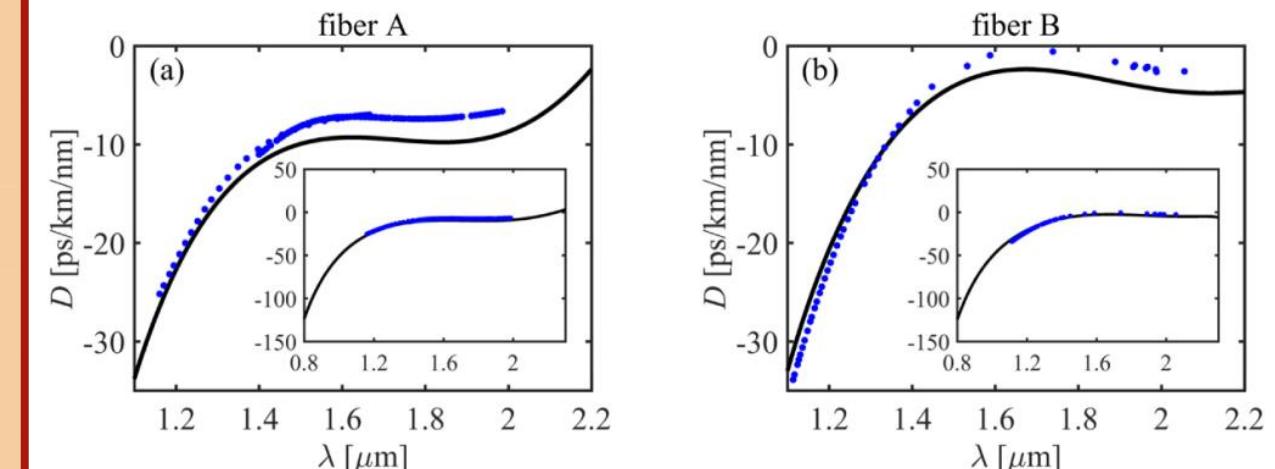
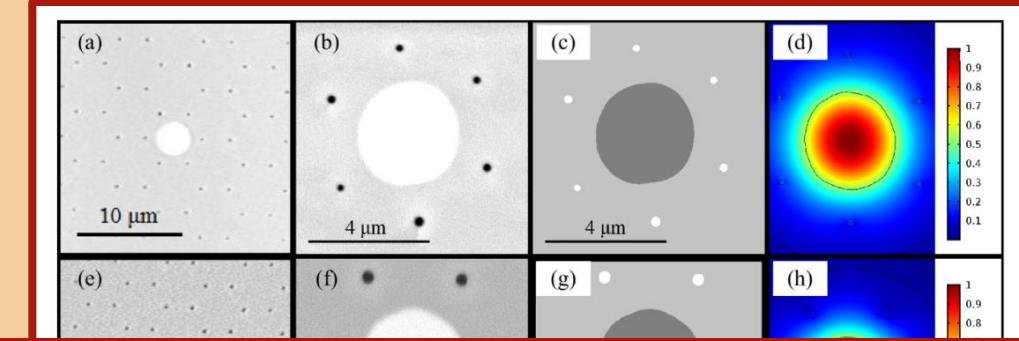
- Discretized conical emission



# All-normal dispersion supercontinuum

Nonlinear microstructured fiber with normal dispersion

- design
- fabrication
- characterization
- supercontinuum generation

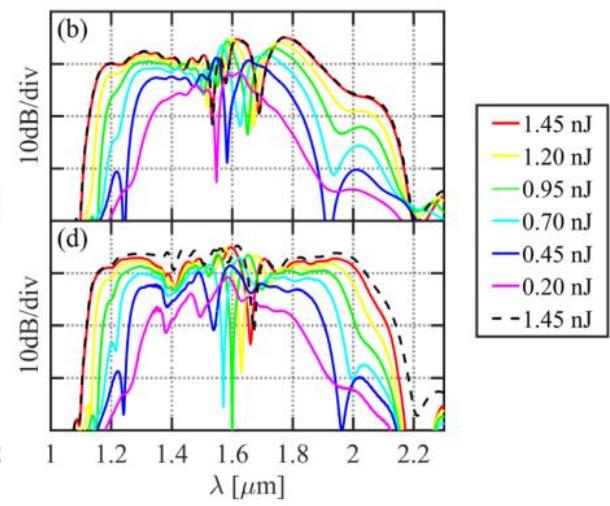
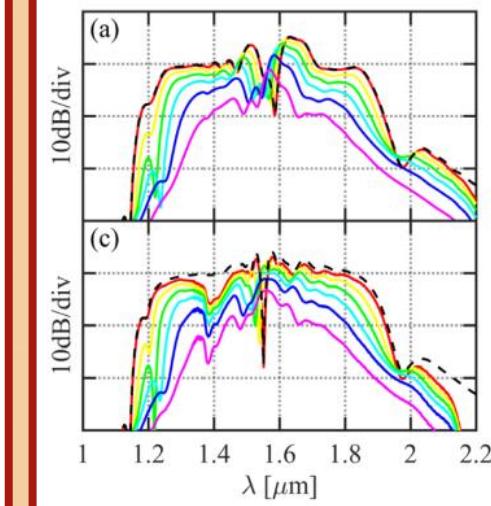
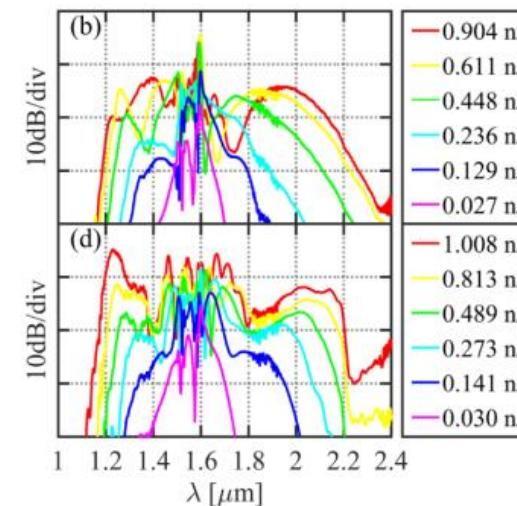
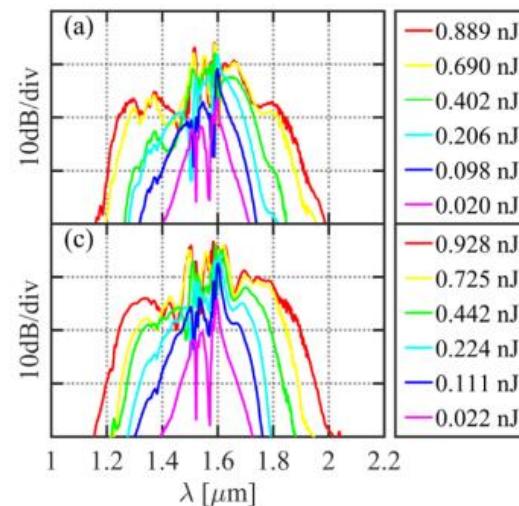


# All-normal dispersion supercontinuum

Nonlinear microstructured fiber with normal dispersion

- broad and coherent supercontinuum spectrum

23-fs  
pumping



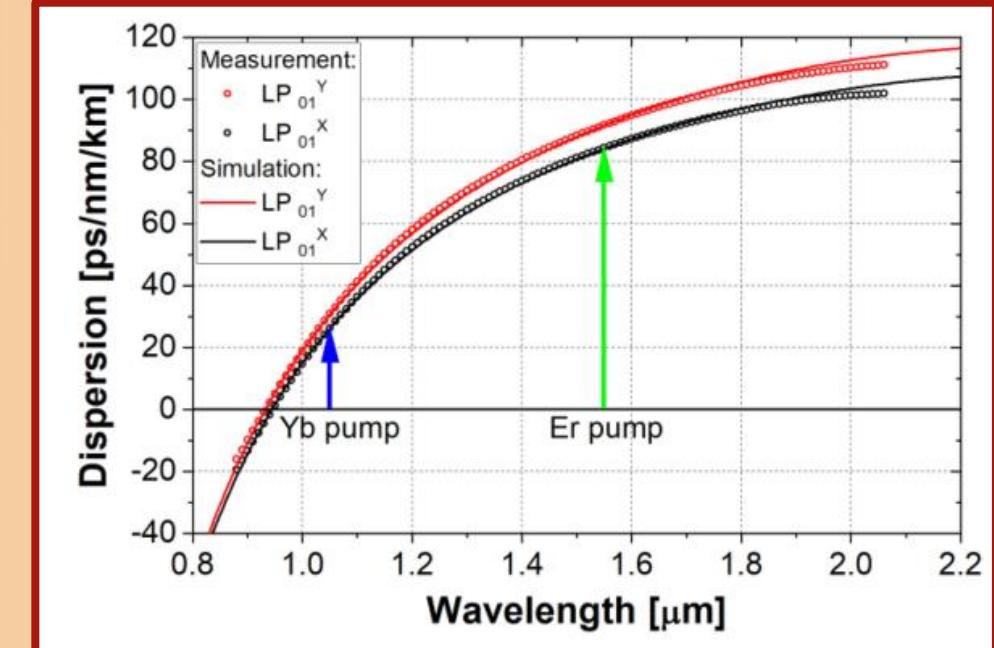
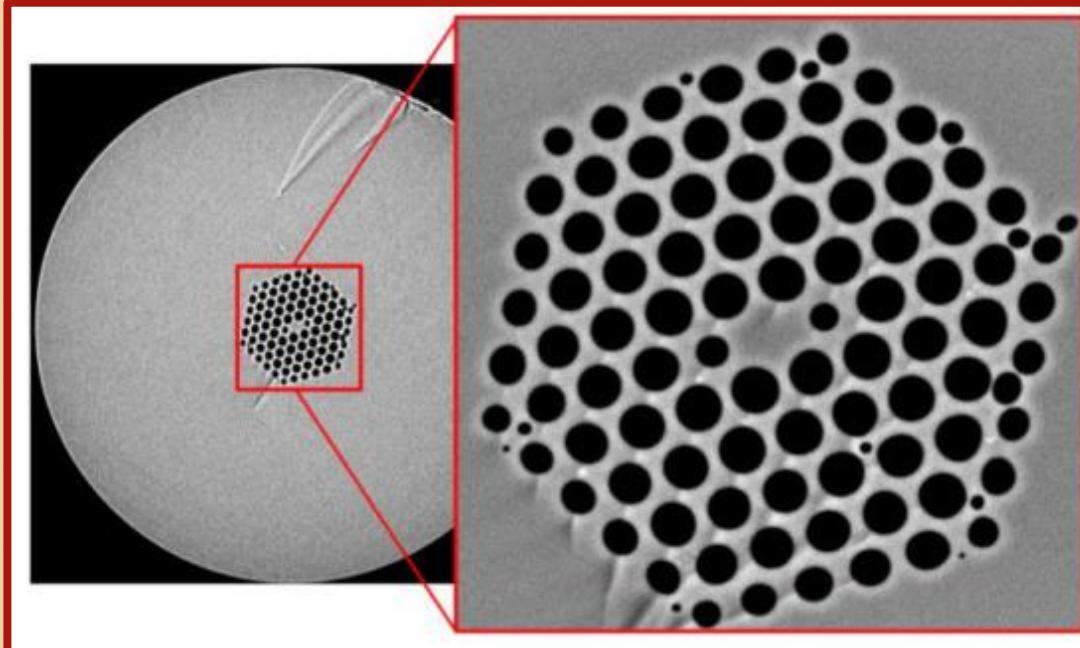


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# Soliton self-frequency shift

Nonlinear microstructured fiber with anomalous dispersion

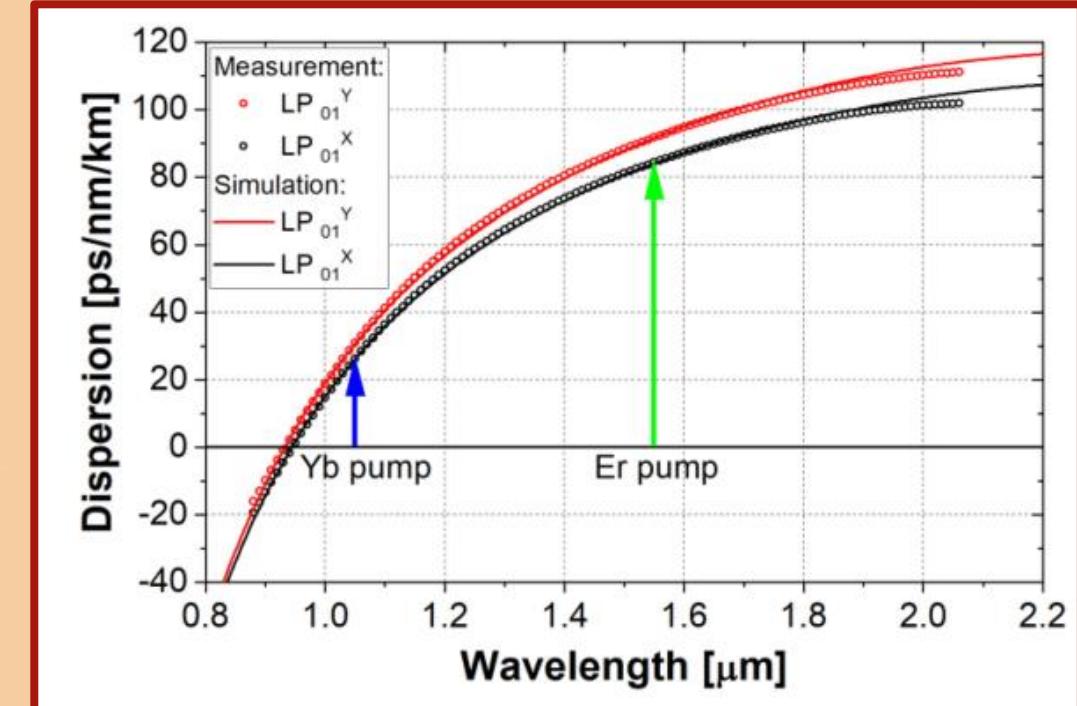
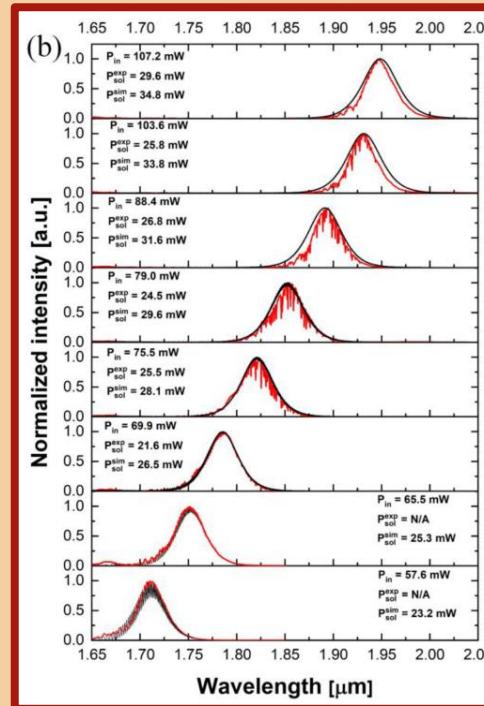
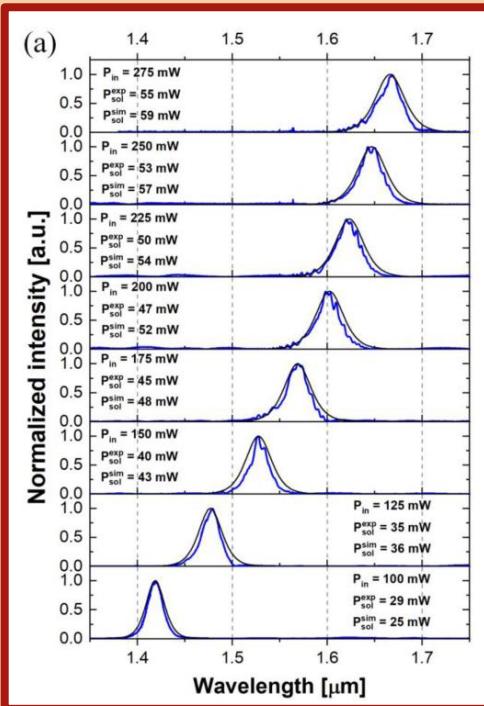
- broad spectral tuning



# Soliton self-frequency shift

Nonlinear microstructured fiber with anomalous dispersion

- broad spectral tuning





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# Polarized all-normal supercontinuum (Pol-AND SC)

## Coupled nonlinear Schrödinger equations

$$\tilde{C}_x = \sqrt[4]{\frac{A_{\text{eff}}(\omega)}{A_{\text{eff}}(\omega_0)}} \tilde{A}_x, \quad \tilde{C}_y = \sqrt[4]{\frac{A_{\text{eff}}(\omega)}{A_{\text{eff}}(\omega_0)}} \tilde{A}_y$$

$$\begin{aligned} \frac{\partial \tilde{C}_x}{\partial z} = & D_x(\tilde{C}_x) + \\ & + i \frac{n_2 n_0 \omega}{c n_{\text{eff}} \sqrt{A_{\text{eff}}(\omega) A_{\text{eff}}(\omega_0)}} \cdot \mathcal{F} \left\{ \left( |C_x|^2 + \frac{2}{3} |C_y|^2 \right) C_x + \frac{1}{3} C_y^2 C_x^* \exp(-2i\Delta\beta z) \right\} \end{aligned}$$

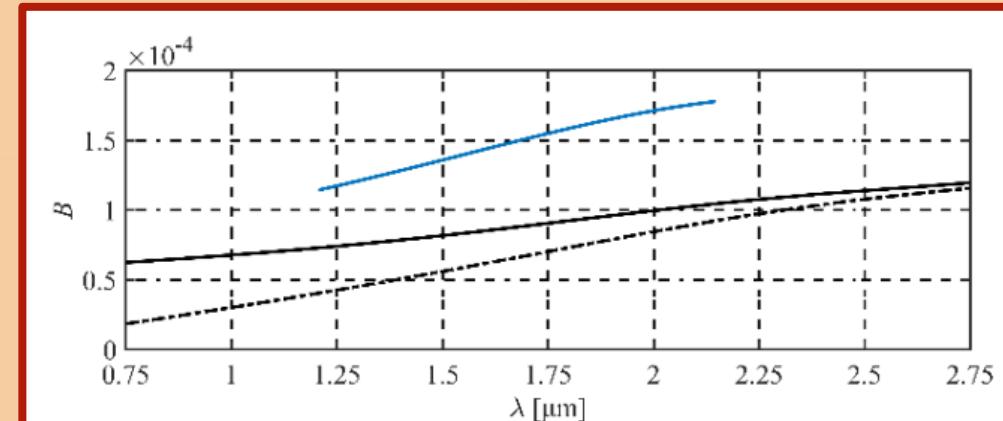
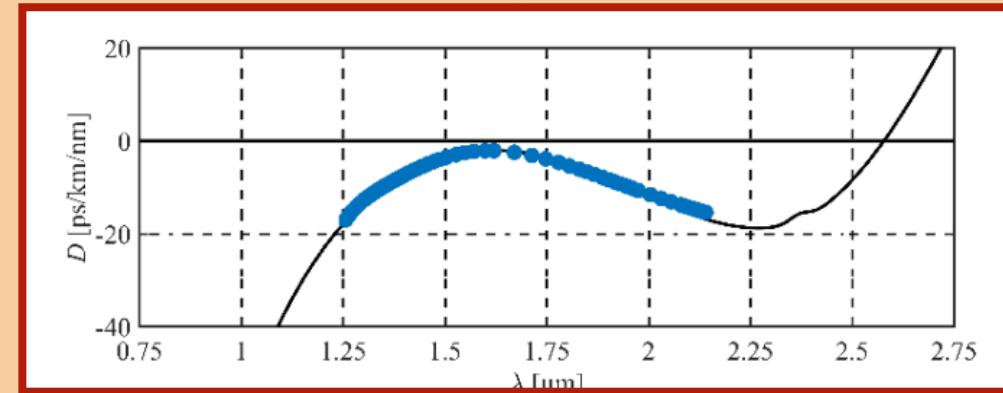
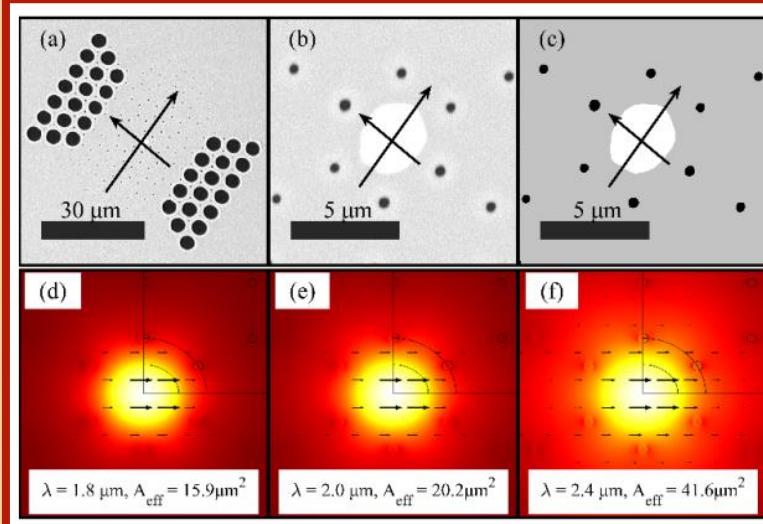
$$\begin{aligned} \frac{\partial \tilde{C}_y}{\partial z} = & D_y(\tilde{C}_y) + \\ & + i \frac{n_2 n_0 \omega}{c n_{\text{eff}} \sqrt{A_{\text{eff}}(\omega) A_{\text{eff}}(\omega_0)}} \cdot \mathcal{F} \left\{ \left( |C_y|^2 + \frac{2}{3} |C_x|^2 \right) C_y + \frac{1}{3} C_x^2 C_y^* \exp(+2i\Delta\beta z) \right\} \end{aligned}$$



# Polarized all-normal supercontinuum (Pol-AND SC)

Nonlinear birefringent microstructured fiber with normal dispersion

- design
- fabrication
- characterization
- supercontinuum generation

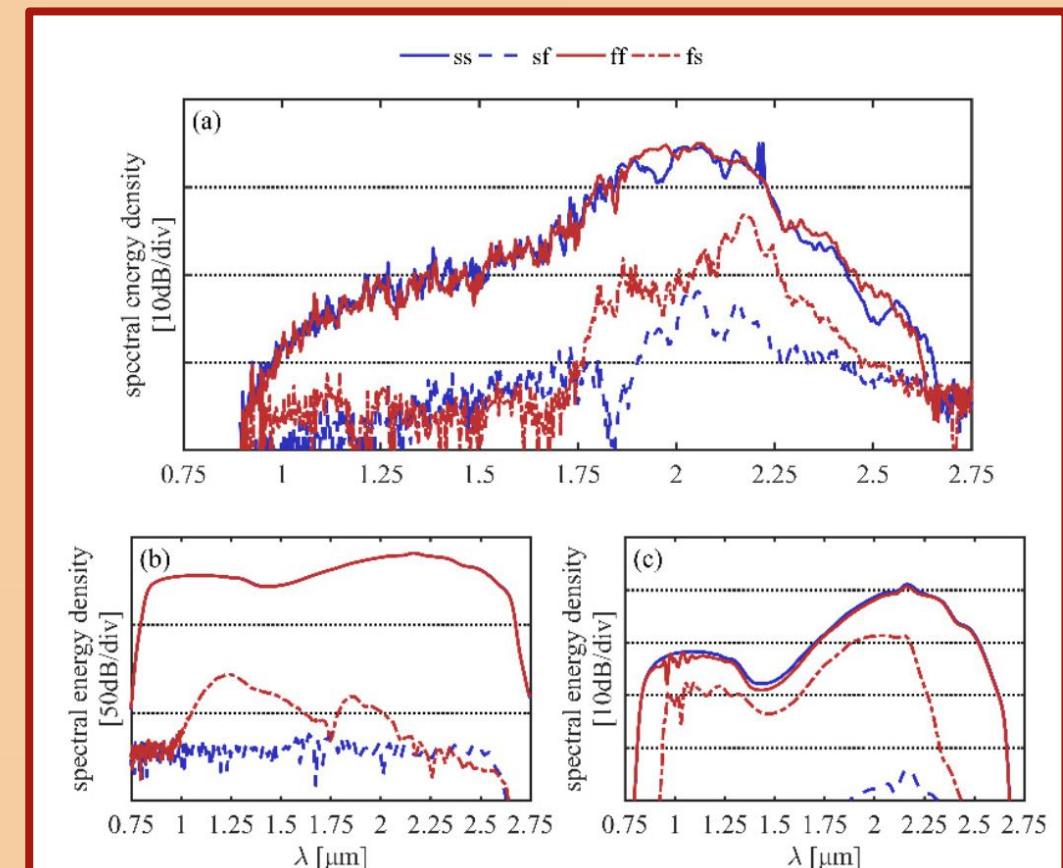




# Polarized all-normal supercontinuum (Pol-AND SC)

## Supercontinuum generation

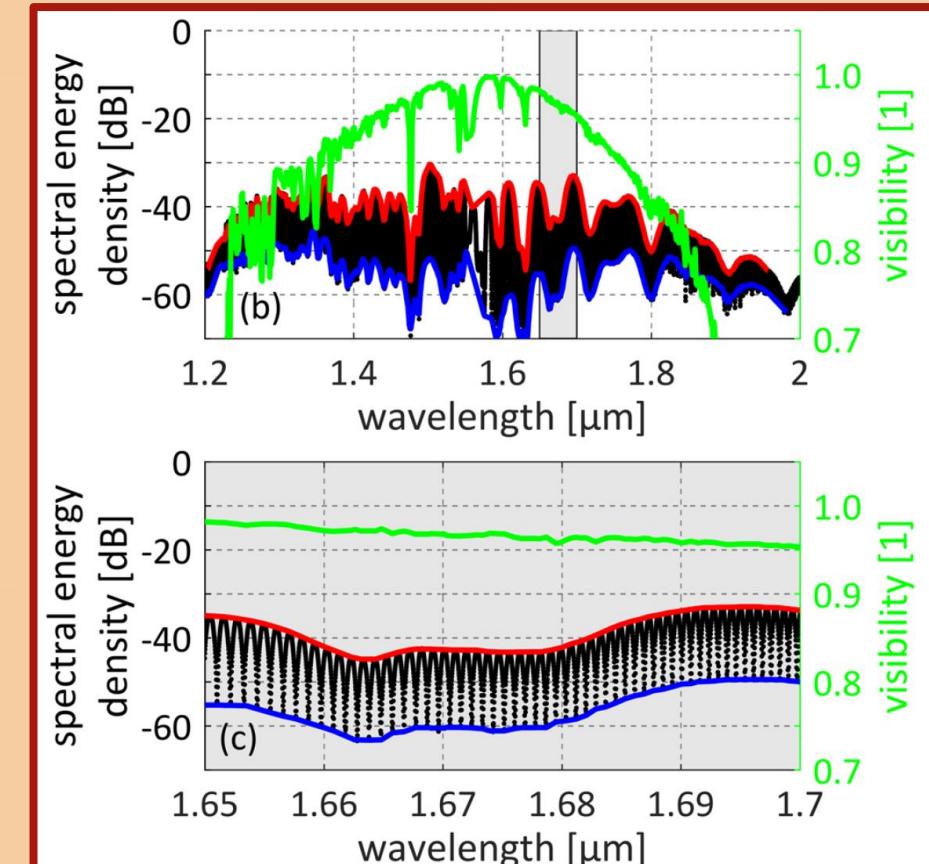
- normal dispersion
- linearly polarized
- coherent



# Polarized all-normal supercontinuum (Pol-AND SC)

## Supercontinuum generation

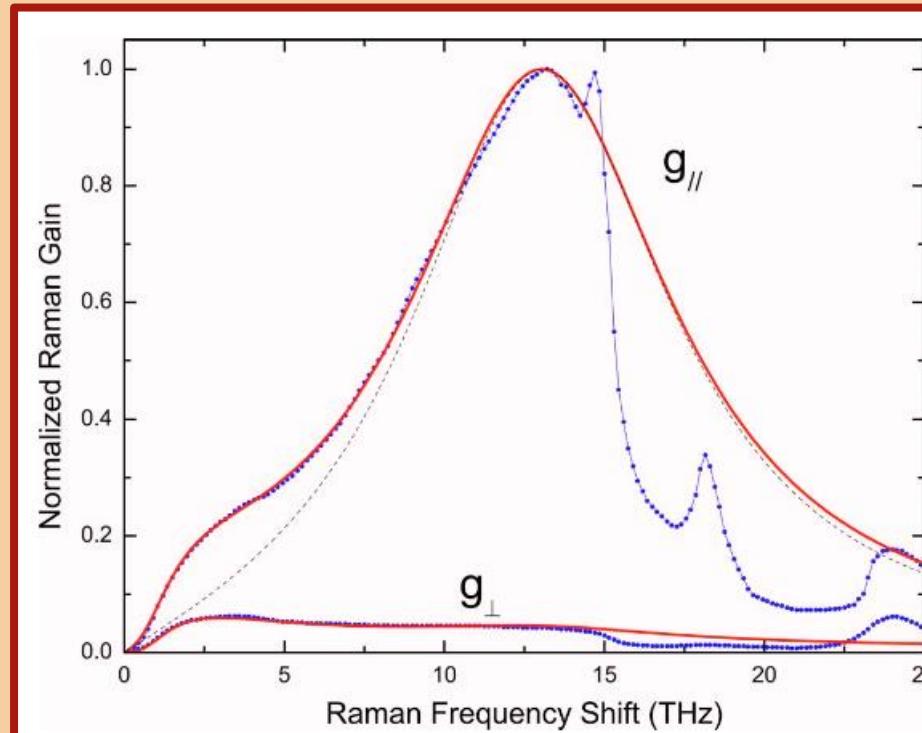
- normal dispersion
- linearly polarized
- coherent





# Orthogonal Raman scattering

Raman response function



Q. Lin, G.P. Agrawal, Optics Letters 31(21): 3086 (2006)



# Orthogonal Raman scattering

Coupled nonlinear Schrödinger equations

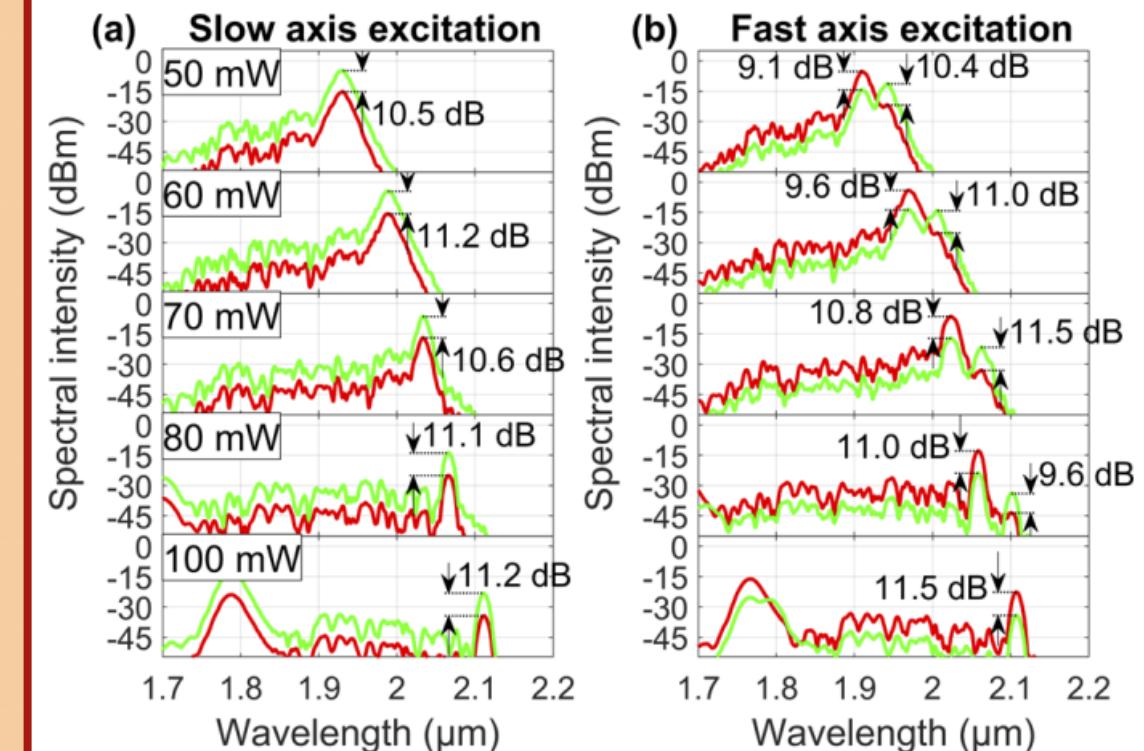
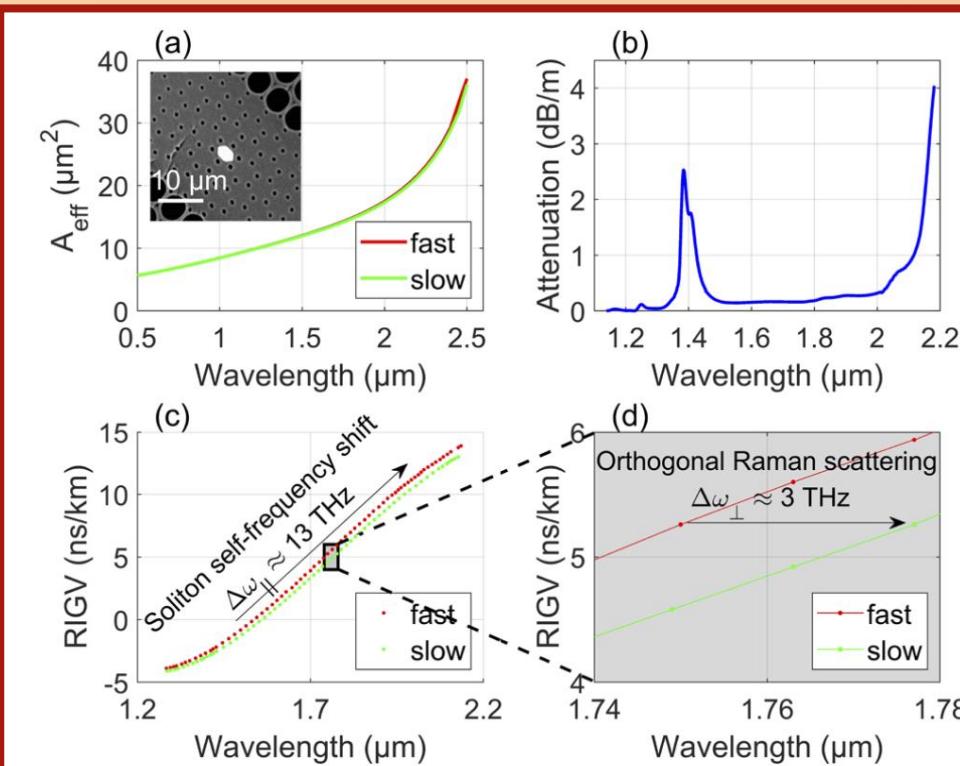
- with vector Raman response

$$N_x(\tilde{C}_x, \tilde{C}_y) = \bar{\gamma}_x \mathcal{F} \left\{ \begin{array}{l} \left(1 - f_R\right) \times \left( \left( |C_x|^2 + \frac{2}{3} |C_y|^2 \right) C_x + \frac{1}{3} C_y^2 C_x^* \exp(-2i\Delta\beta z) \right) + \\ + f_R \times \left[ \begin{array}{l} \left( h_1 \otimes |C_x|^2 + h_2 \otimes |C_y|^2 \right) C_x + \\ + h_3 \otimes (C_x C_y^* + C_y C_x^* \exp(-2i\Delta\beta z)) C_y \end{array} \right] \end{array} \right\}$$

$$N_y(\tilde{C}_y, \tilde{C}_x) = \bar{\gamma}_y \mathcal{F} \left\{ \begin{array}{l} \left(1 - f_R\right) \times \left( \left( |C_y|^2 + \frac{2}{3} |C_x|^2 \right) C_y + \frac{1}{3} C_x^2 C_y^* \exp(+2i\Delta\beta z) \right) + \\ + f_R \times \left[ \begin{array}{l} \left( h_1 \otimes |C_y|^2 + h_2 \otimes |C_x|^2 \right) C_y + \\ + h_3 \otimes (C_y C_x^* + C_x C_y^* \exp(+2i\Delta\beta z)) C_x \end{array} \right] \end{array} \right\}$$

# Orthogonal Raman scattering

## Polarization conversion





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- Far-detuned four-wave mixing

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- Discretized conical emission



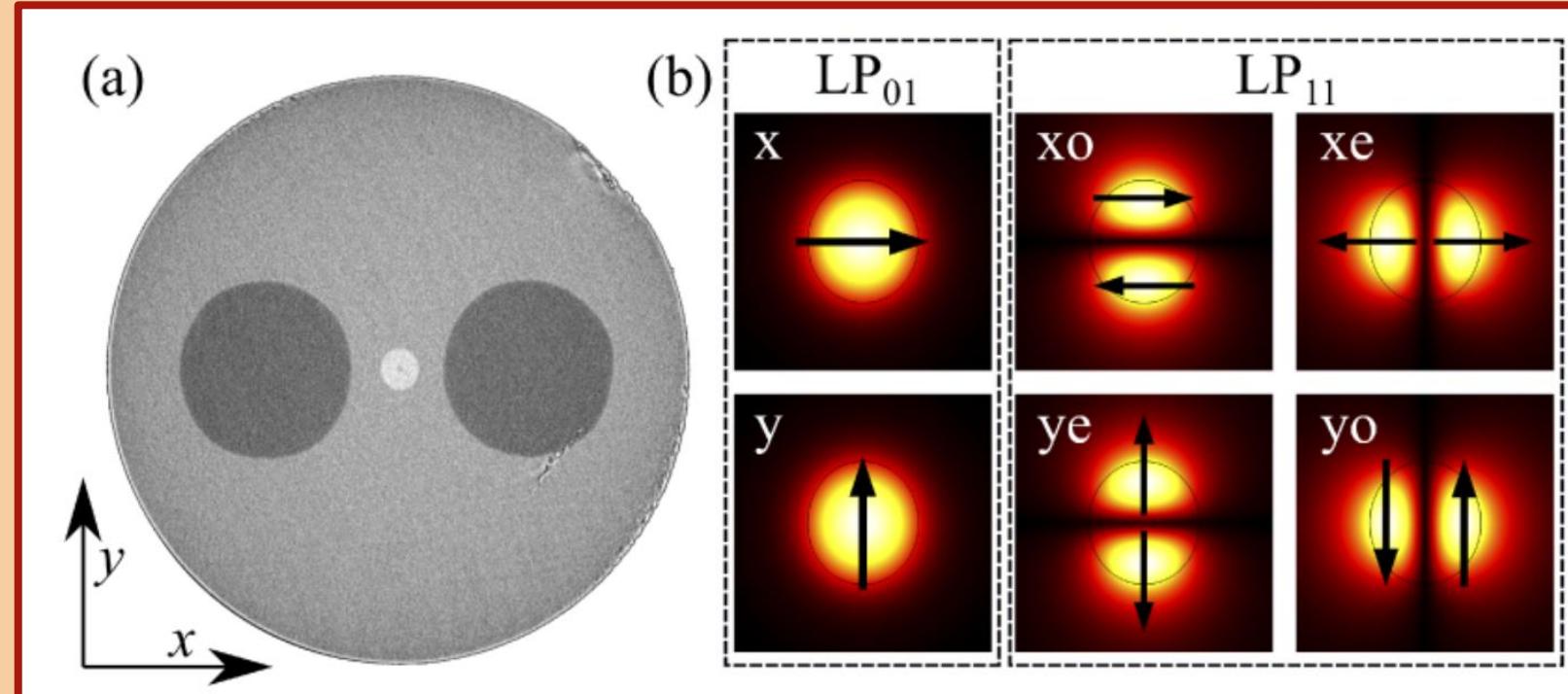
# Few mode fibers

System of nonlinear Schrödinger equations

$$\begin{aligned} \frac{\partial \mathbf{A}_p}{\partial z} = & -\frac{\alpha_p}{2} \mathbf{A}_p + i \left( \beta_0^{(p)} - \beta_0^{(0)} \right) \mathbf{A}_p + \\ & - \left( \beta_1^{(p)} - \beta_1^{(0)} \right) \frac{\partial \mathbf{A}_p}{\partial t} + i \sum_{n=2}^{\infty} \frac{i^n \beta_n^{(p)}}{n!} \frac{\partial^n \mathbf{A}_p}{\partial t^n} + \\ & + i \frac{n_2 \omega_0}{c} \left( 1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \times \\ & \times \sum_{l,m,n}^{N-1} \left\{ \left( 1 - f_R \right) S_K^{(plmn)} \mathbf{A}^{(l)} \mathbf{A}^{(m)} \mathbf{A}^{(n)*} + f_R S_R^{(plmn)} \mathbf{A}^{(l)} \left[ h \otimes \left( \mathbf{A}^{(m)} \mathbf{A}^{(n)*} \right) \right] \right\} \end{aligned}$$

# Intermodal-vectorial FWM

## Fiber modes

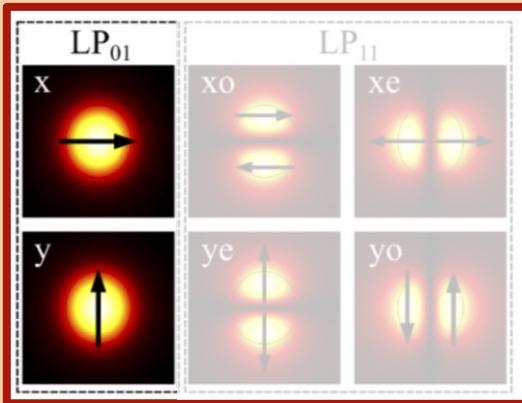


S. Majchrowska, K. Żołnacz et al., Optics Letters 47(10): 2522 (2022)



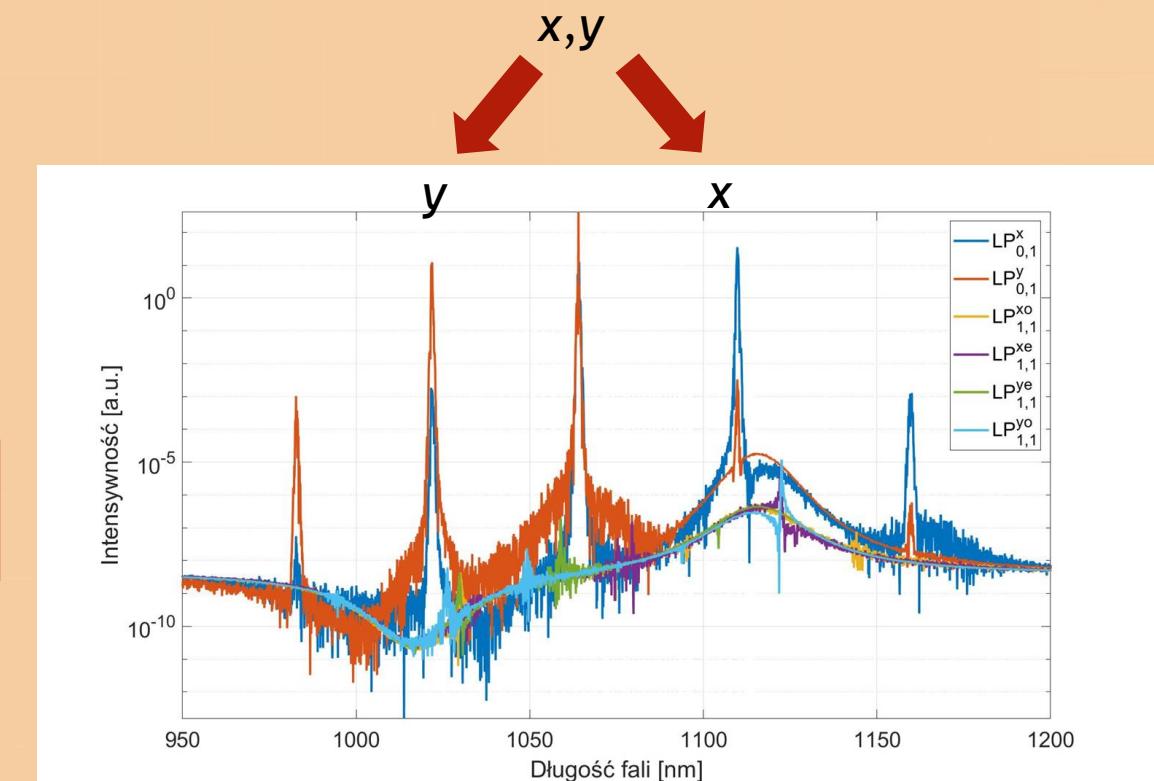
# Intermodal-vectorial FWM

## Vectorial four-wave mixing



$$\beta_0^x + \beta_0^y = \beta_0^x + \beta_1^x \Omega + \frac{1}{2} \beta_2^x \Omega^2 + \beta_0^y - \beta_1^y \Omega + \frac{1}{2} \beta_2^y \Omega^2$$

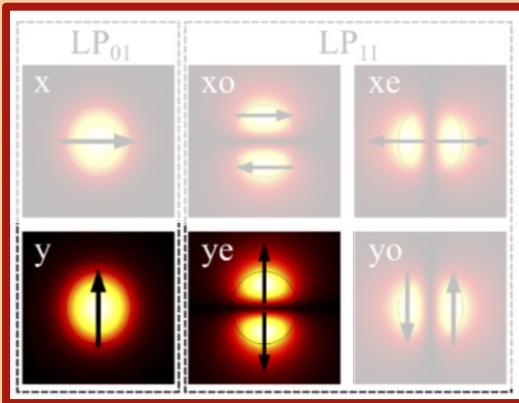
$$-\Delta\beta_1\Omega = \beta_2\Omega^2$$





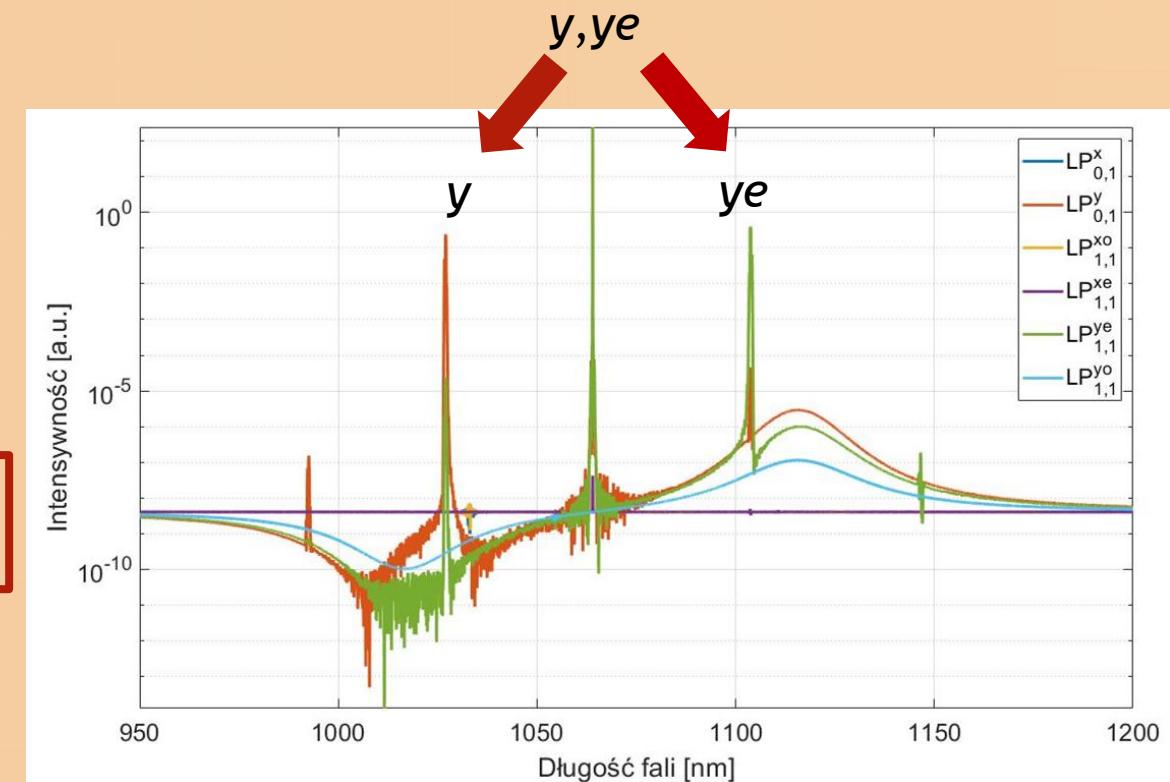
# Intermodal-vectorial FWM

## Intermodal four-wave mixing



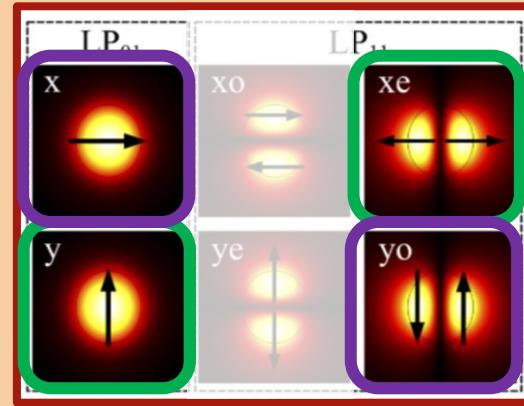
$$\beta_0^y + \beta_0^{ye} = \beta_0^y + \beta_1^y \Omega + \frac{1}{2} \beta_2^y \Omega^2 + \beta_0^{ye} - \beta_1^{ye} \Omega + \frac{1}{2} \beta_2^{ye} \Omega^2$$

$$-\Delta\beta_1 \Omega = \beta_2 \Omega^2$$



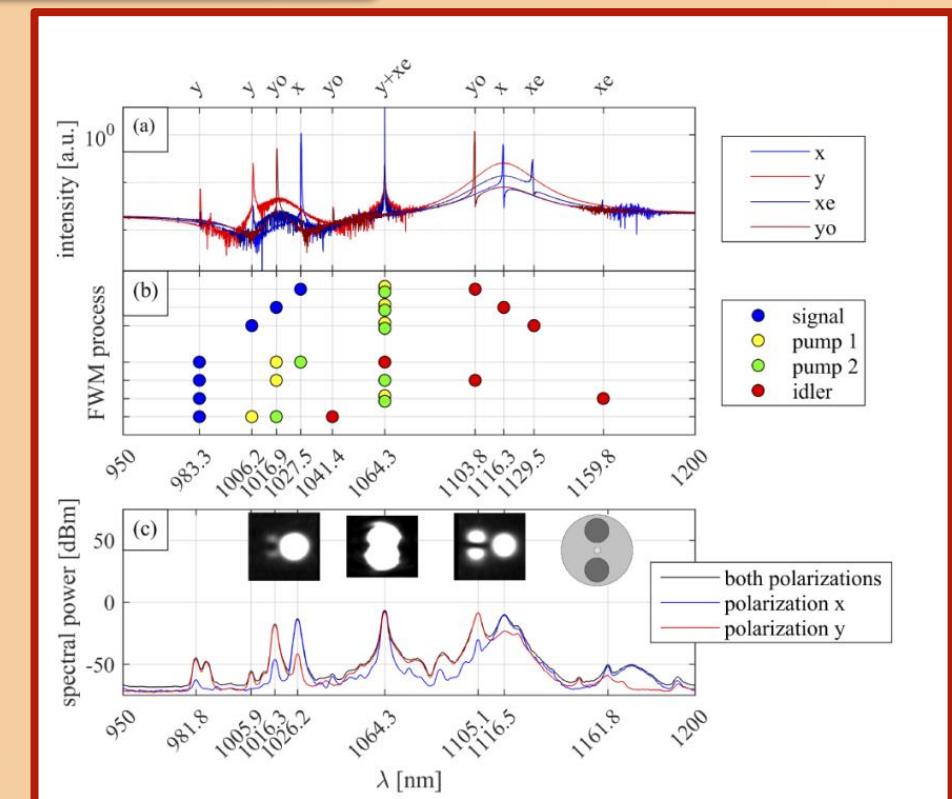
# Intermodal-vectorial FWM

Processes enabled by selective excitation of modes



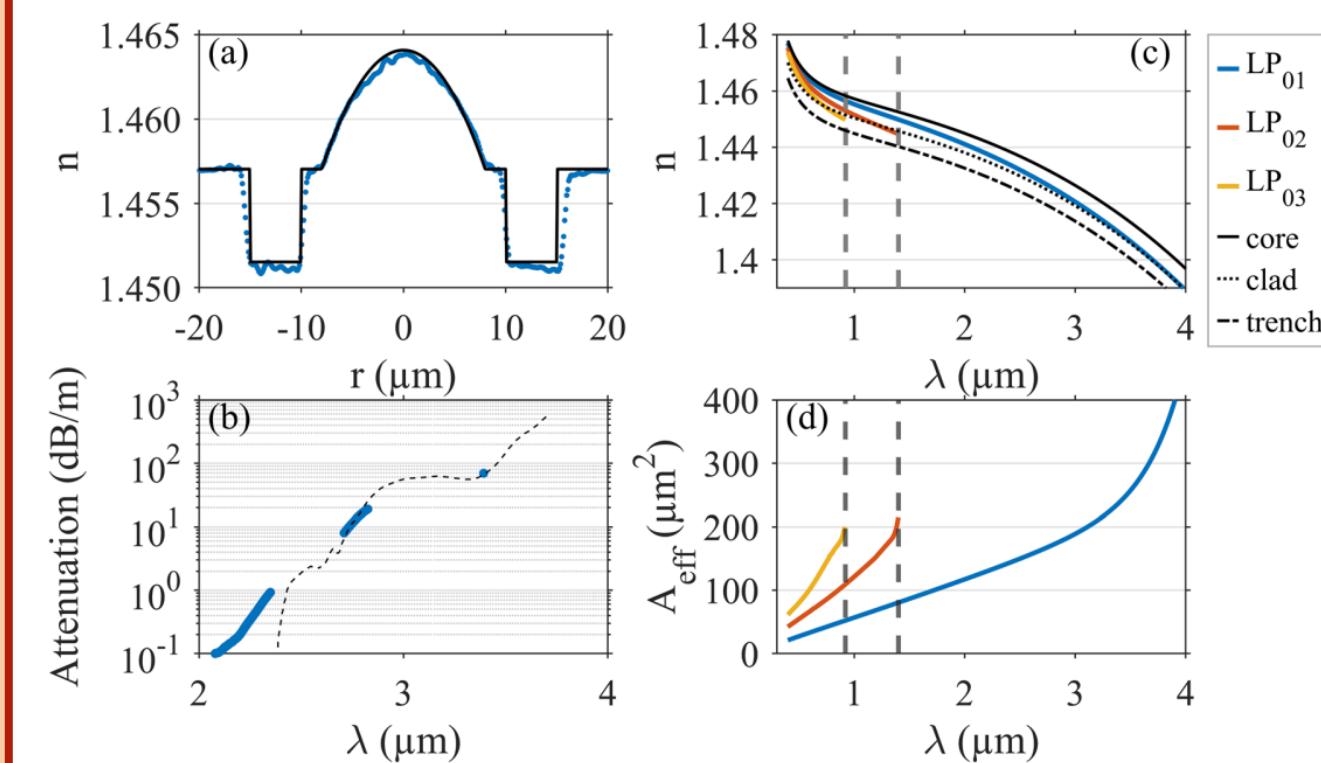
$$\beta_0^y + \beta_0^{xe} = \beta_0^x + \beta_1^x \Omega + \frac{1}{2} \beta_2^x \Omega^2 + \beta_0^{ye} - \beta_1^{ye} \Omega + \frac{1}{2} \beta_2^{ye} \Omega^2$$

$$\frac{1}{2} (\beta_2^x + \beta_2^{ye}) \Omega^2 + (\beta_1^x - \beta_1^{ye}) \Omega + \Delta \beta_0^{x,y} - \Delta \beta_0^{xe,ye} = 0$$



# Far-detuned FWM

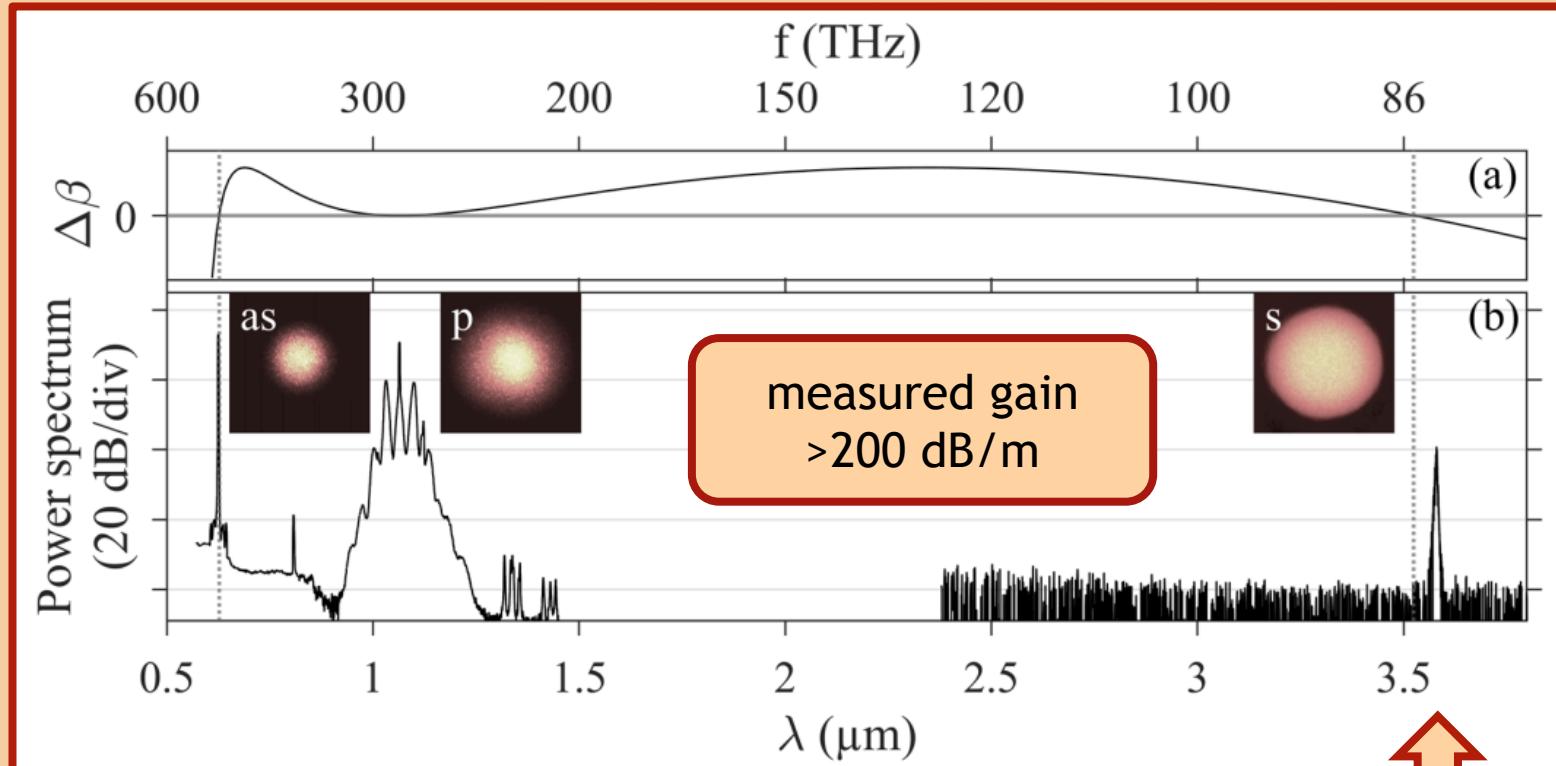
Graded-index fiber





# Far-detuned FWM

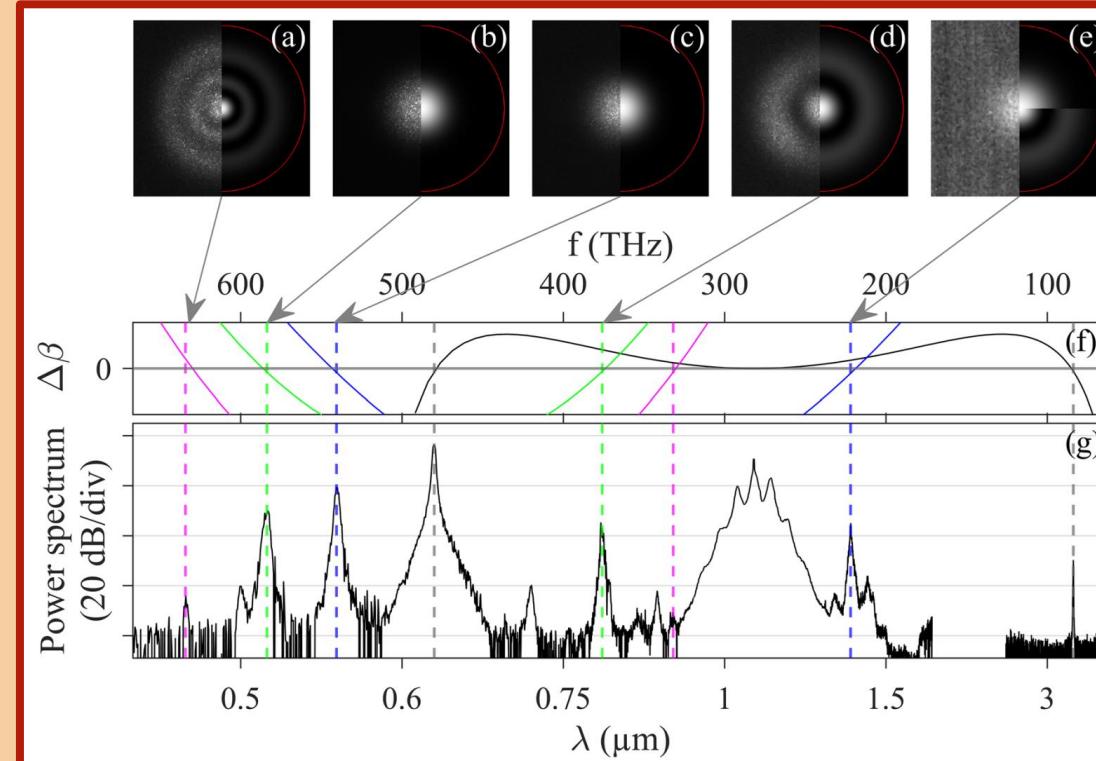
Graded-index fiber





# Far-detuned FWM

Graded-index fiber





# Outline

## Introduction

- Description of frequency conversion processes in optical fibers

## Single mode propagation

- All-normal dispersion supercontinuum
- Soliton self-frequency shift

## Birefringent fibers

- Polarized all-normal dispersion SC
- Solitons - orthogonal Raman scattering

## Few mode fibers

- Intermodal-vectorial four-wave mixing
- Far-detuned four-wave mixing

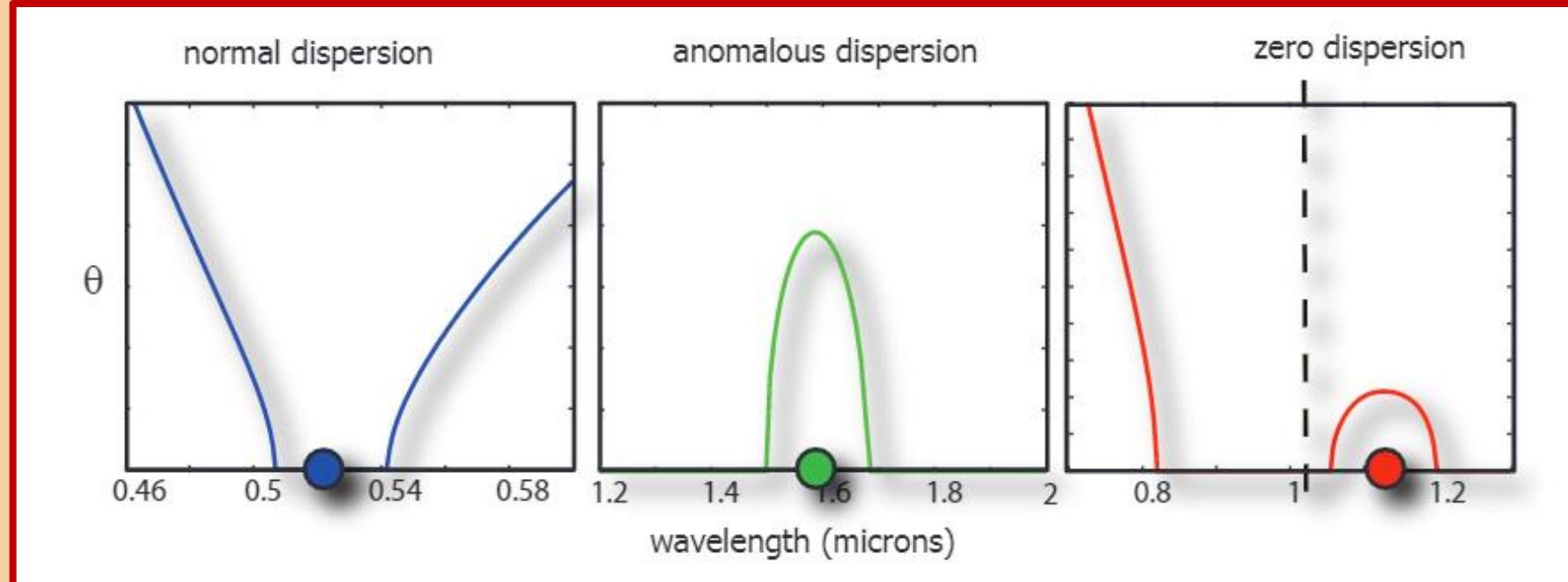
## Multimode fibers

- Discretized conical emission



# Discretized conical emission

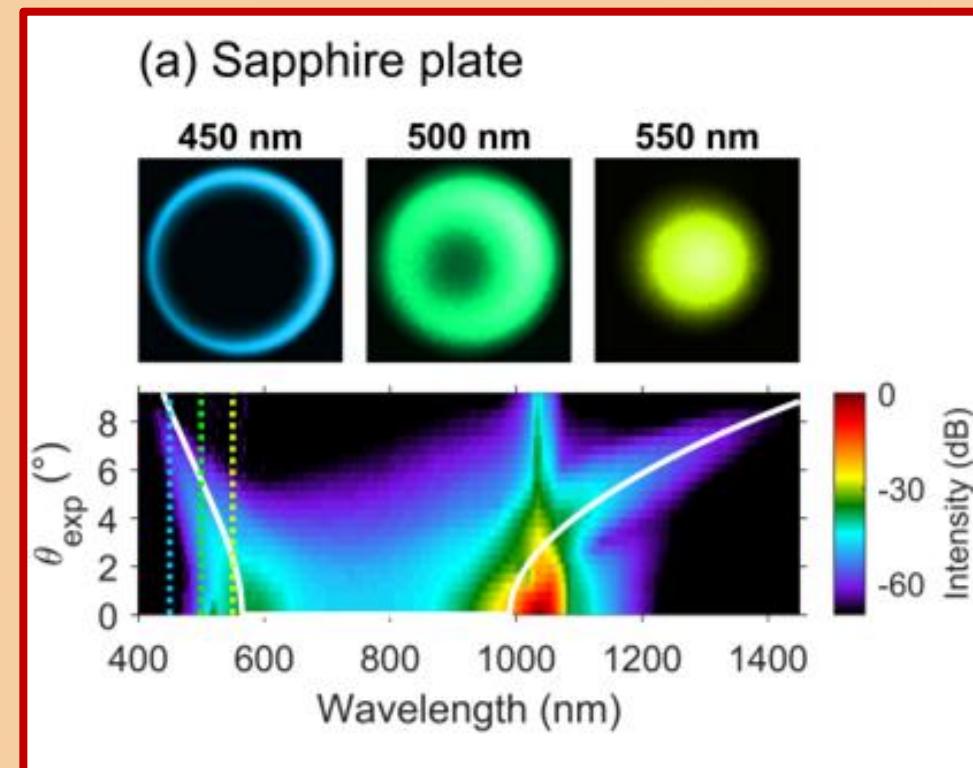
## Conical waves





# Discretized conical emission

Conical emission in bulk

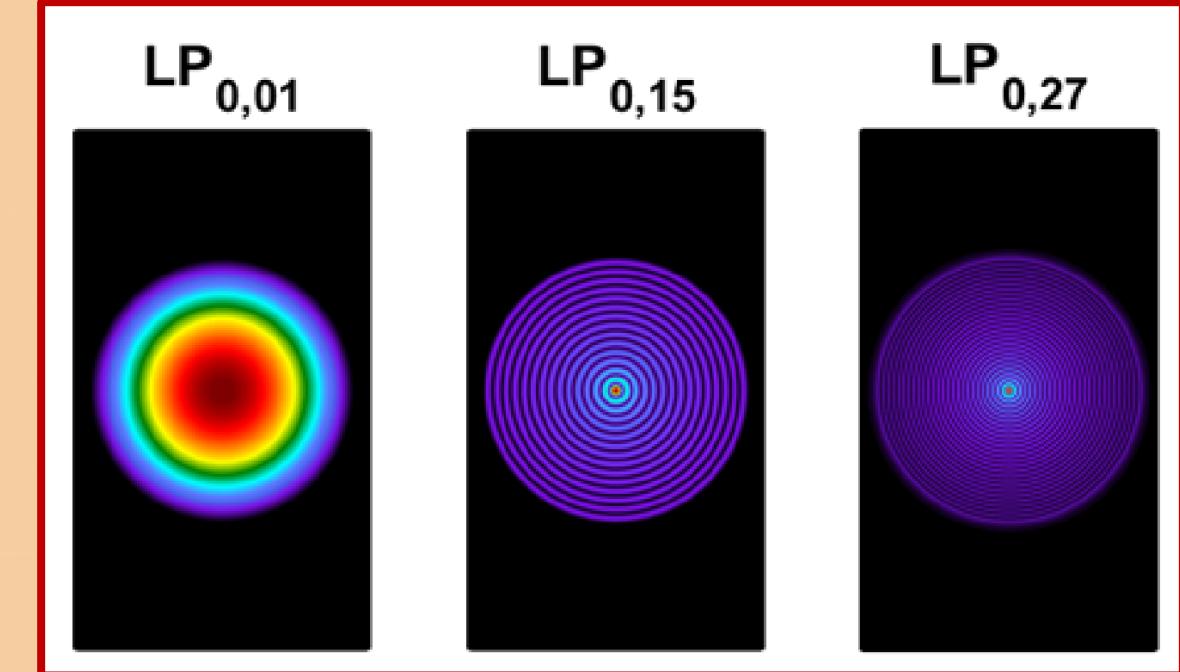




# Discretized conical emission

## Multimode optical fiber

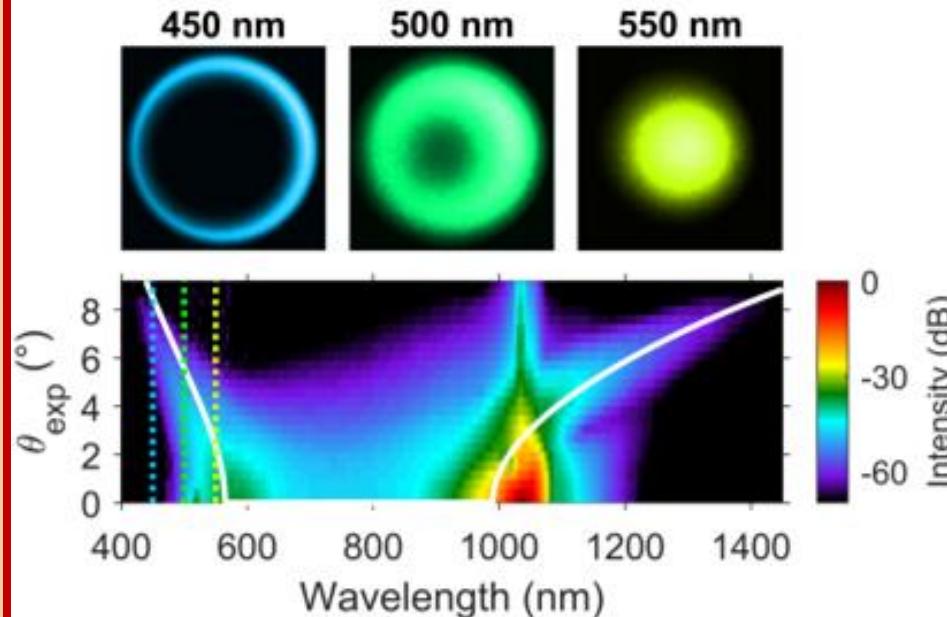
- Core diameter 105  $\mu\text{m}$
- NA = 0.22



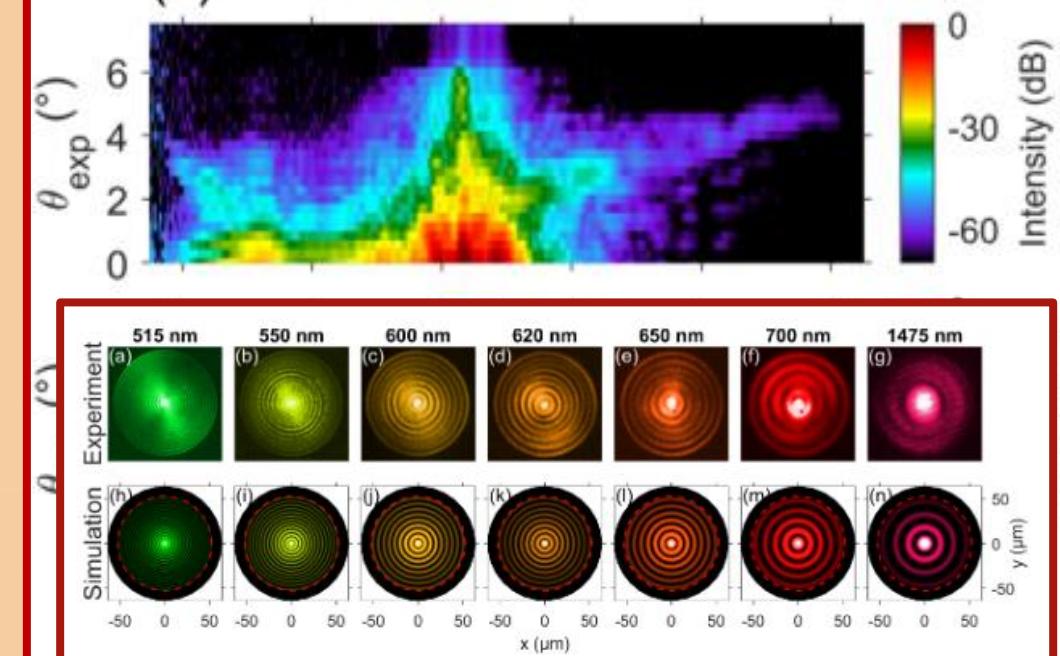
# Discretized conical emission

## Experimental results

(a) Sapphire plate



(b) Multimode fiber





# Conclusions

Optical fibers allow to observe and investigate the broad spectrum of frequency conversion processes

The numerical experiments allow to get insight into the complex dynamics of nonlinear phenomena



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